

FANTASIA *mathematica*

BEING A SET OF STORIES, TOGETHER WITH A
GROUP OF ODDMENTS AND DIVERSIONS,
ALL DRAWN FROM THE UNIVERSE
OF MATHEMATICS



ASSEMBLED AND EDITED,
WITH AN INTRODUCTION, BY

CLIFTON FADIMAN



HOPE AND SCHUSTER, NEW YORK

The Universal Library

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“COME AND sit down over here, Max,” said Professor Wallhausen, “and stop digging around on my desk. I assure you there is nothing there which you could use for your magazine.”

Max Burkel walked over to the living-room table, sat down slowly and reached for his beer glass. “Well, prosit, old boy. Nice to be here again. But no matter what you say, you’ve got to write something for me.”

“Unfortunately I don’t have any good ideas right now. Besides, so much superfluous stuff is being written and, unfortunately, printed too—”

“You don’t have to tell that to a harassed editor like yours truly. The question is, however, just what is superfluous stuff? The authors and their public completely fail to agree about that. Same for editors and reviewers. Well, my three weeks’ vacation is just beginning. In the meantime my assistant can do the worrying.”

“I have sometimes wondered,” said Mrs. Wallhausen, “that you can still find something new for the printer. I should think that by now practically everything that can be expressed with letters has been tried.”

“One would think so, but the human mind seems to be inexhaustible—”

“In repetitions, you mean.”

“Well, yes,” Burkel admitted, “but also when it comes to new ideas and expressions.”

“Just the same,” mused Professor Wallhausen, “one could express

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Authorized translation by Willy Ley

in print everything which can ever be given to humanity, be it historical information, scientific understanding of the laws of nature, poetic imagination and power of expression, or even the teachings of wisdom. Provided, of course, that it can be expressed in words. After all, our books conserve and disseminate the results of thought. But the number of the possible combinations of a given number of letters is limited. Therefore all possible literature must be printable in a finite number of volumes."

"Dear friend," said Burkel, "now you are talking as a mathematician rather than as a philosopher. How can all possible literature, including that of the future, make a finite number of books?"

"I'll figure out in a moment how many volumes would be required to make a Universal Library. Will you—" he turned to his daughter—"hand me a sheet of paper and a pencil from my desk?"

"Bring the logarithm table too," Burkel added dryly.

"Not necessary, not necessary at all," the professor declared. "But now our literary friend has the first word. I ask: If we are frugal and do away with various fonts of type, writing only for a hypothetical reader who is willing to put up with some typographical inconveniences and who is only interested in the meaning—"

"There is no such reader," Burkel said firmly.

"I said 'hypothetical reader.' How many different characters would one need for printing general literature?"

"Well," said Burkel, "let's just stick to the upper- and lower-case letters of the Latin alphabet, the customary punctuation marks and the space that keeps the words apart. That wouldn't be too much. But for scientific works, that's another story. Especially you mathematicians have an enormous number of symbols."

"Which could be replaced by an agreement with small indices like a_1, a_2, a_3 and a^1, a^2 and a^3 , adding just twice ten characters. One could even use this system to write words from languages which do not use the Latin alphabet."

"All right. Maybe your hypothetical, or better, your ideal reader will put up with that too. Under these conditions we could probably express everything with, say one hundred different characters."

"Well, well. Now how big do you want each volume?"

"I should think that one can exhaust a theme pretty well with five hundred book pages. Let's say that there are forty lines per page and

fifty characters per line, we'll have forty times fifty times five hundred characters per volume, which is—you calculate it."

"One million," said the professor. "Therefore, if we take our one hundred characters, repeat them in any order often enough to fill a volume which has room for one million characters, we'll get a piece of literature of some kind. Now if we produce all possible combinations mechanically we'll ultimately get all the works which ever have been written in the past or can be written in the future."

Burkel slapped his friend's shoulder. "You know, I'm going to subscribe right now. This will furnish me with all the future volumes of my magazine; I won't have to read manuscripts any more. This is wonderful for both editor and publisher: the elimination of the author from the literary business! The replacement of the writer by the automatic printing press! A triumph of technology!"

"What?" said Mrs. Wallhausen. "You say everything will be in that library? The complete works of Goethe? The Bible? The works of all the classical philosophers?"

"Yes, and with all the variations of wording nobody has thought up yet. You'll find the lost works of Tacitus and their translations into all living and dead languages. Furthermore, all of my and friend Burkel's future works, all forgotten and still undelivered speeches in all parliaments, the official version of the Universal Declaration of Peace, the history of the subsequent wars, all the compositions all of us wrote in school and college—"

"I wish I had had this volume when I was in college," Mrs. Wallhausen said. "Or would it be volumes?"

"Volumes, probably. Don't forget that the space between words is a typographical character too. A book may contain a single line; everything else might be empty. On the other hand, even the longest works could be accommodated because if they don't fit into one volume they could be continued through several."

"No, thanks. Finding something must be a chore."

"Yes, this is one of the difficulties," Professor Wallhausen said with a pleased smile, looking after the smoke from his cigar. "At first glance one should think that this would be simplified by the fact that the library must contain its own catalogue and index—"

"Good!"

"The problem would be to find that one. Moreover, if you had found an index volume it wouldn't help you any because the con-

tents of the Universal Library are not only indexed correctly, but also in every possible incorrect and misleading manner."

"The devil! But unfortunately true."

"Yes, there would be quite a number of difficulties. Let's say we take the first volume of the Universal Library. Its first page is empty, so is the second and the third and so forth through all five hundred pages. This is the volume in which the 'space' has been repeated one million times."

"At least that volume can't contain any nonsense," Mrs. Wallhausen observed.

"Hardly a consolation. But we'll take the second volume. Also empty until, on page 500, line 40 at the extreme end, there is a lonely little 'a.' Same thing in the third volume, but the 'a' has moved up one place. And then the 'a' slowly moves up, place by place, through the first million volumes until it reaches the first place on page 1, line 1 of the first volume of the second million. Things continue that way through the first hundred million volumes until each of the hundred characters has made its lonely way from last to first place in the books. The same then happens with 'aa' or with any other two characters. One volume could contain one million period marks and another one million question marks."

"Well," said Burkel, "it should be simple to recognize and discard those volumes."

"Maybe, but the worst is yet to come. It happens when you have found a volume which seems to make sense. Say you want to refresh your memory about a passage in Goethe's *Faust* and you manage to locate a volume with the right beginning. But when you have progressed for a page or two it goes on 'aaaaaa' and that is the only thing in the remaining pages of the book. Or you find a table of logarithms, but you can't tell whether it is correct. Remember, the Universal Library contains everything which is correct, but also everything which is not. You can't trust the chapter headings either. A volume may begin with the words 'History of the Thirty Years War' and then say: "After the nuptials of Prince Blücher and the Queen of Dahomey had been celebrated at Thermopylae"—you see what I mean. Of course, nobody would ever be embarrassed. If an author has written the most incredible nonsense it will, of course, be in the Universal Library. It will be under his by-line. But it will also be under the by-line of William Shakespeare and under any other possible by-line. He

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will find one of his books where it is asserted after every sentence that all this is nonsense and another one where it is stated after the self-same sentences that they are the purest wisdom."

"I have enough," said Burkel. "I knew as soon as you started that this was gong to be a tall tale. I won't subscribe to your Universal Library. It would be impossible to sift truth from falsehood, sense from nonsense. If I find several million volumes all claiming to be the true history of Germany during the twentieth century and all contradicting each other, I would do much better by reading the original works of the historians."

"Very clever! Otherwise you would have taken on an impossible burden. But I wasn't telling a tall tale in your sense. I did not claim that you could use the Universal Library, I merely said that it is possible to tell exactly how many volumes would be required for a Universal Library containing all possible literature."

"Go ahead and calculate," said Mrs. Wallhausen; "it is easy to see that this blank sheet of paper bothers you."

"Not needed," said the professor; "that I can do in my head. All we have to do is to realize very clearly how that library is going to be produced. First we put down each one of our hundred characters. Then we add to each, every one of our hundred characters, so that we have one hundred times one hundred groups of two characters each. Adding the third set of our hundred characters we get $100 \times 100 \times 100$ groups of three characters each and so forth. Since we have one million possible positions per volume, the total number of volumes is 100 raised to the millionth power. Now since 100 is the square of 10, we obtain the same figure if we write a '10' with two million as the power. This is simply a '1' followed by two million zeros. Here it is: $10^{2,000,000}$."

"You make your life easy," remarked Mrs. Wallhausen. "Why don't you write it down in the normal manner?"

"Not me. This would take me at least two weeks, without time out for food and sleep. If you printed that figure, it would be a little over two miles long."

"What is the name of that figure?" the daughter wanted to know.

"It has no name. There isn't even a way in which we could hope to grasp that figure, it is so colossal, even though it is finite."

"How about expressing it in trillions?" asked Burkel.

"A mathematical trillion is a nice big figure, a '1' followed by 18

zeros. But if you express the number of volumes in trillions, you get a figure with 1,999,982 zeros instead of two million zeros. It's no help; one is as ungraspable as the other. But, just wait a moment." The professor scribbled a few figures on the sheet of paper.

"I knew it would come to that!" Mrs. Wallhausen said with much satisfaction.

"All done," her husband announced. "I assumed that each volume is two centimeters thick and that the whole library is arranged in one long row. How long do you think this row would be?"

"I know," said the daughter. "You want me to say it?"

"Go ahead."

"Twice as many centimeters as the number of volumes."

"Bravo, my dear. Absolutely correct. Now let's look at this more closely. You know that the speed of light, expressed in metric units, is 300,000 kilometers per second, which for a year amounts to about 10,000 million kilometers, which equals 1,000,000,000,000,000,000 centimeters, your mathematical trillion, Burkel. If our librarian can move with the speed of light it will still take him two years to pass a trillion volumes. To go from one end of the library to the other with the speed of light will take twice as many years as there are trillions of volumes in the library. We had that figure before, and I feel that nothing shows more clearly how impossible it is to grasp the meaning of this $10^{2,000,000}$ even though, as I have said repeatedly, it is a finite figure."

"If the ladies will permit, I have one more question," said Burkel. "I suspect that you have calculated a library for which there is no room in the universe."

"We'll see in a moment," the professor answered, reaching for the pencil. "Well, I assumed you packed the library in 1000-volume boxes, each box having a capacity of precisely one cubic meter. All space to the farthest known spiral galaxies would not hold the Universal Library. In fact, you would need this volume of space so often that the number of packed universes would be a figure with only some 60 zeros less than the figure for the number of volumes. No matter how we try to visualize it, we are bound to fail."

"I thought all along that it was infinite," said Burkel.

"No, that's just the point. The figure is not infinite, it is a finite figure. The mathematics of it are flawless. What is surprising is that we can write down on a very small piece of paper the number of

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volumes comprising all possible literature, something which at first glance seems to be infinite. But if we then try to visualize it—for example, try to find a specific volume—we realize that we cannot grasp what is otherwise a very clear and logical thought which we evolved ourselves.”

“Well,” concluded Burkel, “coincidence plays, but reason creates. And for this reason you’ll write down tomorrow what amused us tonight. That way I’ll get an article for my magazine which I can carry with me.”

“All right. I’ll write it down for you. But I’m telling you right now that your readers will conclude that this is an excerpt from one of the superfluous volumes of the Universal Library.”